The Effect of Mass Transfer on Horizontal Boundary-Layer Flows with Combined Free and Forced Convection

WILLIAM N. GILL and EDUARDO DEL CASAL

Syracuse University, Syracuse, New York

The effect of mass transfer, or finite interfacial velocities, on the velocity and temperature fields for the case neglecting gravitational effects has been rather extensively investigated and is referenced and discussed in a number of textbooks (*1*, *2*). Eichhorn (*3*) has studied the problem in vertical, free-convection boundary-layer flows. The present work considers horizontal boundary-layer flows where the coupling between the energy and momentum equations is different from that in vertical flows.

In a previous paper (*4*) exact solutions were given for the velocity and temperature fields for horizontal, nonisothermal flow past a flat plate with gravity effects included. Here, the influence of mass transfer across the plate-fluid interface will be studied. The notation used in the previous paper is retained in identical form.

When one uses the equations and similarity substitutions determined previously for flow over a horizontal flat plate, the present problem requires solutions to the momentum equation

$$f''''(\eta) + f'(\eta)\, f''(\eta) + f(\eta)\, f'''(\eta) + N_2\, [\theta_2(\eta) + \eta\, \theta_2'(\eta)] = 0 \quad (1)$$

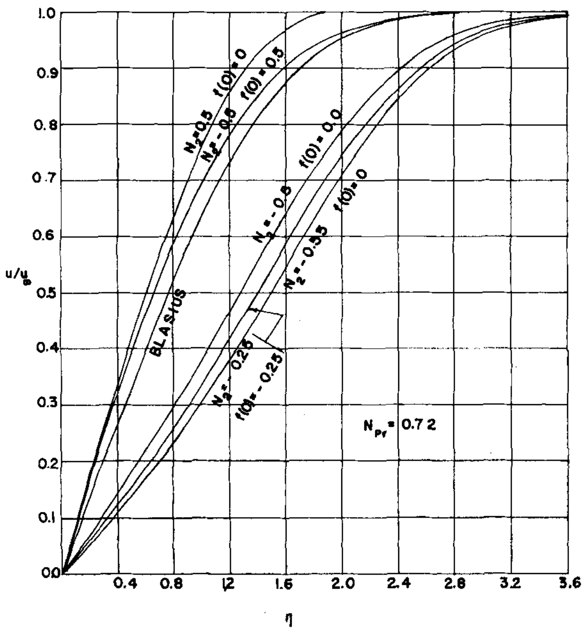

Fig. 1. Velocity profiles for horizontal boundary-layer flows with combined free and forced convection.

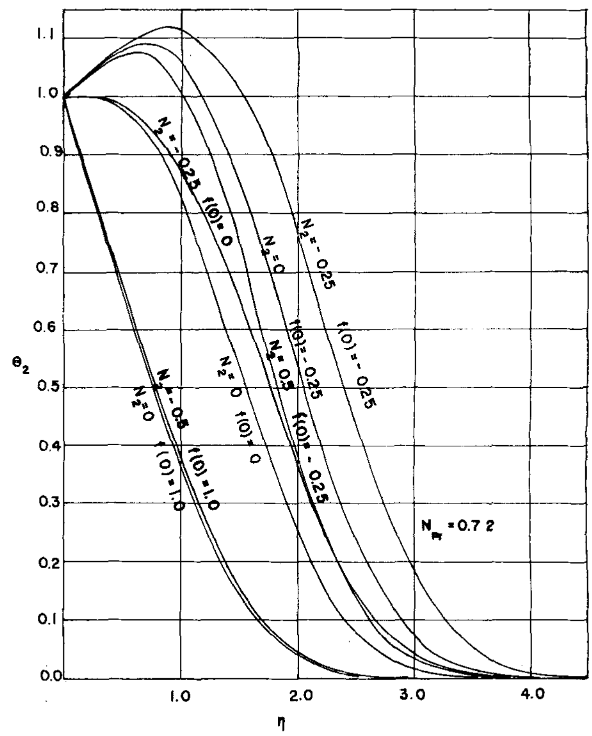

Fig. 2. Temperature profiles for horizontal boundary-layer flow with combined free and forced convection.

INFORMATION RETRIEVAL*

Key Words: Fluid Flow-8, Flow-8, Poiseuille-, Knudsen-, Laminar Flow-8, Gases-9, Capillaries-9, Equations-9, Kinetic Theory-10, Predictions-8, Flow-9, Solids-9, Porosity-9, Permeability-9, Physical Properties-9, Properties (Characteristics)-9, Design-8, Vacuum-9, Piping-9.

Abstract: A new equation for the flow of gases in capillaries is presented, in which all flow constants can be calculated from the simple kinetic theory of gases. The equation is shown to reproduce experimental results over the entire range of laminar, slip, and Knudsen flow within the expected accuracy. Applications to the design of vauum piping and flow through porous solids are discussed.

Reference: Scott, D. S., and F. A. L. Dullien, A.I.Ch.E. Journal, **8**, No. 3, p. 293 (July, 1962).

(Continued on page 424)

* For details on the use of these key words and the A.I.Ch.E. Information Retrieval Program, see *Chem. Eng. Progr.*, **57**, No. 5, p. 55 (May, 1961), No. 6, p. 73 (June, 1961)

(*Continued from page 423*)

INFORMATION RETRIEVAL*

Key Words: Properties (Characteristics)-9, Physical Properties-9, Transport-9, Gases-9, Fluids-9, Solids-9, Porosity-9, Flow-8, Fluid Flow-8, Porosity-8, Vacuum-8, Freezing-8, Drying-8, Heat Transfer-8, Food-5, Solids-5, Pressure-6, Conductivity-7, Permeability-7, Freezers-10, Dryers-10.

Abstract: Theory is outlined relative to the effect of pressure on permeabilities and thermal conductivities of gases in porous solids in the slip flow and free-molecule flow regions, and experimental results are reported for several freeze-dried food materials. Mean pore diameters calculated from permeability data are in agreement with microscopic observations; measured thermal conductivities follow the predicted behavior of a constant value at high pressures, with a gradual decrease to another constant value at very low pressures.

Reference: Harper, John C., **A.I.Ch.E. Journal, 8,** No. 3, p. 298 (July, 1962).

Key Words: Flow-8, Fluid Flow-8, Heat Transfer-8, Transferring-8, Equations-9, Reynolds Number-6, Eigenvalues-7, Temperature-7, Distribution-7, Theories-10, Prandtl-, Numerical-10, Methods-10, Techniques-10.

Abstract: Heat transfer in transition flow has been studied analytically, with the results of a modified Prandtl mixing length theory used. Eigen-values, Nusselt numbers, and temperature distributions have been obtained for flows between constant temperature parallel plates at Reynolds numbers from 4,800 to 43,000 and Prandtl numbers from 0.01 to 100. Local Nusselt numbers and thermal entrance lengths also have been computed. The practicality of four numerical methods used in solving the equations of interest is discussed.

Reference: Gill, William N., and Shaw Mei Lee, **A.I.Ch.E. Journal, 8,** No. 3, p. 303 (July, 1962).

Key Words: Mass Transfer-8, Rates-8, Vaporization-8, Evaporation-8, Absorption-8, Distillation-8, Interfaces-9, Areas (Surface)-9, Packings-9, Correlations-9, Water-1, Methanol-1, Ethanol-1, Alcohols-1, Acetone-1, Keytones-1, Toluene-1, Hydrocarbons-1, Trichloroethylene-1, Chlorinated Hydrocarbons-1, Halogenated Hydrocarbon-1, Air-5, Water-5, Surface Tension-6, Physical Properties-6, Properties (Characteristics)-6, Flow Rates-6, Interfaces-7, Areas (Surface)-7, H.T.U.-7, Towers-10, Columns (Process)-10, Packed-, Raschig Rings-10, Rings-10.

Abstract: Mass transfer rates have been studied in columns packed with Raschig rings for three kinds of operations: vaporization of water into air, absorption of methanol vapor by water, and distillation of three binary mixtures—trichloroethylene-toluene, ethanol-water, and acetone-water. Correlations are presented for the individual phase H.T.U.'s and also for the effective interfacial areas. The effective areas for a given packing vary depending on the kind of operation as well as on the liquid flow rate and surface tension.

Reference: Yoshida, Fumitake, and Tetsushi Koyanagi, **A.I.Ch.E. Journal, 8,** No. 3, p. 309 (July, 1962).

Key Words: Flow-9, Fluid Flow-9, Fluid Mechanics-9, Laminar Flow-9, Rheology-8, Liquid Phase-8, Polymers-8, Solutions (Mixtures)-8, Shear-8, Stresses-8, Equations-8, Tubes-10, Cylinders-10, Pipes-10,

Abstract: Equilibrium flow of a general fluid through a long straight cylindrical tube is examined, and equations are derived for determining the three pertinent material functions: the shear stress component, the difference between the radial and angular normal stress components, and the difference between the axial and angular normal stress components. These equations are expressed in terms of quantities which are measurable. Experimental data have been obtained for a polymer solution and the material functions calculated.

Reference: Sakiadis, B. C., **A.I.Ch.E. Journal, 8,** No. 3, p. 317 (July, 1962).

(*Continued on page 426*)

and the energy equation

$$\theta_2''(\eta) + N_{Pr}\,[f(\eta)\,\theta_2'(\eta) + f'(\eta)\,\theta_2(\eta)] = 0 \quad (2)$$

where f and θ_2 are the dimensionless stream function and temperature distribution respectively, and

$$\eta = \frac{1}{2}\frac{\left(\frac{y\,u_\infty}{\nu}\right)}{\left(\frac{x\,u_\infty}{\nu}\right)^{1/2}}$$

The boundary conditions are

$$f'(0) = f''(\infty) = 0;\ f'(\infty) = 2 \quad (3)$$

$$f(0) = \text{constant}$$

and

$$\theta_2(0) = 1;\ \theta_2(\infty) = 0 \quad (4)$$

The condition of arbitrary, but constant, $f(0)$ enables one to determine the combined influence of mass transfer and the gravitational field on the flow.

When one integrates, Equation (1) gives

$$f''' + ff'' + N_2\,\eta\,\theta_2 = f''(0) + f(0)\,f'(0) \quad (5)$$

and Equation (2) becomes

$$\theta_2 = e^{-N_{Pr}\int_0^\eta f d\eta} \quad (6)$$

Equation (6) results, since it can be shown with the boundary conditions given in Equations (4) that

$$\theta'_2(0) = -N_{Pr}\,f(0)$$

which yields the heat flux directly. Also, with Equations (3), it follows that

$$f''(0) + f(0)\,f'(0) = 0$$

and hence Equation (5) becomes

$$f''' + ff'' + N_2\,\eta\,\theta_2 = 0 \quad (7)$$

Let

$$\phi = f''$$

and by partial integration

$$\int_0^\eta f\,d\eta = \eta\,f(0) + \int_0^\eta \frac{(\eta-\sigma)^2}{2}\,\phi(\sigma)\,d\sigma \quad (8)$$

Consequently the same iterative method used previously (*1*) is applicable.

Figure 1 shows several velocity distributions for $N_{Pr} = 0.72$ and various values of the parameters $f(0)$ and N_2. Qualitatively similar deviations from the Blausius profile are caused individually by comparable values of the mass transfer parameter $f(0)$ and the gravitational field parameter N_2 in the range -0.5 to 0.5.

The interaction of N_2 and $f(0)$ for $N_{Pr} = 0.72$ may be seen somewhat more clearly in Table 1, where $\phi(0)$ is tabulated for various $f(0)$ and N_2. If these data are plotted, it is seen that near the stagnation point $\phi(0) = 0$ negative N_2 cause $\phi(0)$ to vary rapidly with $f(0)$. This type of behavior is not observed with mass transfer alone.

Table 1 gives the values of $\phi(0)$ for $N_{Pr} = 0.72$ and all the N_2, $f(0)$ values for which the calculation converged. The numerical calculation requires one

TABLE 1. CALCULATED VALUES OF THE VELOCITY GRADIENT AT THE WALL FOR VARIOUS N_2 AND $f(0)$ AND $N_{Pr} = 0.72$

N_2	$f(0)$	$\phi(0)$	N_2	$f(0)$	$\phi(0)$
0.000	1.000	2.917	0.500	0.000	1.705
0.000	0.500	2.092	0.500	−0.250	1.431
0.000	0.250	1.701	0.500	−0.500	1.217
0.000	0.000	1.328	0.500	−1.000	0.900
0.000	−0.250	0.979	−0.250	1.000	2.823
0.000	−0.500	0.658	−0.250	0.500	1.952
0.000	−1.000	0.143	−0.250	0.250	1.521
0.250	1.000	3.007	−0.250	0.000	1.082
0.250	0.500	2.222	−0.250	−0.250	0.562
0.250	0.250	1.690	−0.250	−0.350	—
0.250	0.000	1.529	−0.500	1.000	2.727
0.250	−0.250	1.233	−0.500	0.500	1.800
0.250	−0.500	0.982	−0.500	0.250	1.312
0.250	−1.000	0.633	−0.500	0.000	0.701
0.500	1.000	3.095	−0.500	−0.150	—
0.500	0.500	2.346			
0.500	0.250	2.009			

to iterate on ϕ until $\phi(0)$ is determined to the desired accuracy. However, as seen in Equation (8), $\int_0^\eta f\, d\eta$ will be negative for larger η if ϕ is negative in the region near the wall. This effect accumulated with successive iterations, and the expression for $\phi(0)$ yields increasingly large negative values. Therefore if $\phi(0)$ is negative, the iterative procedure employed diverges; also, as N_2 approaches the value required for stagnation, the number of iterations needed increases markedly. The exact values of $\phi(0)$, with various N_2, required for stagnation were not determined owing to the excessive numerical work required. However these may be estimated by extrapolating the data in Table 1 graphically and noting the smallest values of the parameters used in the calculation for which $\phi(0)$ is negative.

Figure 2 shows several representative temperature profiles. Clearly, the effect of N_2 is more pronounced for negative $f(0)$ or when there is mass transfer into the stream.

ACKNOWLEDGMENT

This work was sponsored by the National Science Foundation under grant G 15016.

LITERATURE CITED

1. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, "Transport Phenomena," p. 608, Wiley, New York (1960).
2. Eckert, E. R. G., and R. M. Drake, "Heat and Mass Transfer," McGraw Hill, New York (1959).
3. Eichhorn, R., *Trans. Am. Soc. Mech. Engrs., Series C*, **82**, 260 (1960).
4. Gill, W. N., and Eduardo del Casal, *A.I.Ch.E. Journal*, to be published.

Application of Reciprocal Variational Principles to Laminar Flow in Uniform Ducts

WARREN E. STEWART

University of Wisconsin, Madison, Wisconsin

The usefulness of variational principles in solving steady flow problems is well known; a variety of Newtonian and non-Newtonian problems have been so treated (*1, 10, 11, 12, 13, 14*). The problem of determining the accuracy of variational flow calculations however has not received much attention. The present note shows how such a determination can be made, with reciprocal variational principles developed by Hill (*5, 6*) and independently by Johnson (*7, 8*).

The variational theorems of Hill and Johnson are quite general with regard to system geometry, rheological behavior, and boundary conditions. To focus attention on the reciprocal aspects of the theorems the present discussion deals only with rectilinear Newtonian flow, and the basic equations are specialized accordingly. The notation and terminology here parallel those used by Johnson.

BASIC EQUATIONS

Consider the steady flow of a Newtonian fluid, of constant viscosity and density, in a long, cylindrical duct of arbitrary cross section S_0 under a known pressure gradient. The flow is assumed to be parallel to the z-axis, except for a small region at each end of the duct. Determine Q, the volumetric rate of flow.

For this system the equations of continuity and motion (*2*) become

$$\frac{\partial v_z}{\partial z} = 0 \quad (1)$$

$$0 = -\frac{\partial \mathcal{P}}{\partial x} \quad (2a)$$

$$0 = -\frac{\partial \mathcal{P}}{\partial y} \quad (2b)$$

$$0 = -\frac{\partial \mathcal{P}}{\partial z} - \frac{\partial \tau_{xz}}{\partial x} - \frac{\partial \tau_{yz}}{\partial y} \quad (2c)$$

The nonvanishing components of the viscous stress tensor are given by

$$\tau_{xz} = \tau_{zx} = -\mu \frac{\partial v_z}{\partial x} \quad (3a)$$

$$\tau_{yz} = \tau_{zy} = -\mu \frac{\partial v_z}{\partial y} \quad (3b)$$

and the boundary conditions are

$$\text{at } z = 0, \quad \mathcal{P} = \mathcal{P}_0 \quad (4a)$$

$$\text{at } z = L, \quad \mathcal{P} = \mathcal{P}_L \quad (4b)$$

$$\text{at the walls, } v_z = 0 \quad (5)$$

where the region $0 < z < L$ lies inside the region of fully developed flow.

Hill (*5, 6*) and Johnson independently (*7, 8*) have given two methods for variational solution of this problem. By applying both methods one obtains upper and lower bounds on the quantity

$$J_0 = -\frac{1}{2}(\mathcal{P}_0 - \mathcal{P}_L)Q \quad (6)$$